DIRECT ESTIMATION OF NONLINEAR AERODYNAMIC COEFFICIENTS

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Abstract

In this paper a parameter estimation technique is proposed which allows for direct updates to the aerodynamic coefficients represented in the form of table lookup. The proposed method is capable of capturing the nonlinearities in the coefficients. It is also possible to capture joint dependence of the coefficient on one or more independent variables. The estimation is cast in the form of linear least-squares problem. The recursive form of this algorithm is also proposed. The results are demonstrated using simulated data as well as flight data

1. Introduction

Aerodynamic data of an aircraft is updated by Flight-testing. The response of the aircraft to control inputs is used to compute the dynamic model of the aircraft. Estimation of nonlinear dependence of tlie aerodynamic coefficients with respect to the variables like Mach number. angle of attack, sideslip, angular rates and control effectors is a particularly important aspect of the update process. The mathematical model is validated by comparing outputs obtained by simulation with flight data.

There are many techniques developed over the years for estimation of flight characteristics [1,9]. Most of these the techniques result in estimation of the linear equivalent model of the aircraft. This model is valid only in a restricted region of the flight envelope. The linear model can prove useful in quickly establishing the level of stability and for handling qualities evaluations [2]. Tlie linear model assumes particular importance for the calculation of the gain and phase margins for an unstable aircraft [3]. Linear least squares is an efficient tool for the estiniation of flight derivatives. The recursive version of least squares has been shown to be computationally efticient and can be used in real time [4]. Many aircraft do not possess lineal- characteristics. This exposes two limitations of the linear model approach for such aircraft. Firstly, the estimated derivatives are capable of giving only the general trend of the coefficient without capturing the variations.

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Secondly, since linearity of response is restricted to small signal amplitudes only; this presents a problem in choosing the proper level of input excitation. At higher input levels, the nonlinearities dominate the response, while at very low levels estimation accuracy is compromised due to low response levels. This is undesirable panicularly when the stability margins or loss of control effectiveness are to be estimated from flight data.

The wind tunnel model of an aircraft can be quite comprehensive. This is achieved by representing the nonlinear variations in the form of Table lookup. Even with linear interpolation, it is possible to capture the characteristics with sufficient fidelity by choosing break points at proper locations. The linear estimation techniques do not permit direct update of these tables because they capture the average trend rather than local variations.

Techniques for determination of airplane model structure from flight data using splines and modified stepwise regression are available in technical papers [5]. Wind tunnel predictions through incremental coefficients obtained from flight data analysis are presented in [6,7]. Also, postulating suitable derivative models in an analytical form can carry out identification of an aerodynamic database [8]. In this paper we propose an estimation technique. which directly updates the aerodynamic coefficient dependency on one or more independent variables represented in table lookup form. This paper is organized as follows. In Section 2, a nonlinear model of the aircraft short period dynamics is formulated. The simulation response of this system is used to generate data for the estimation problem. Section 3 reforniulates the multi-dimensional linear interpolation problem in a more general form. The procedure for estimation of the table data is posed as least squares estimation of nonlinear characteristics in Section 3. Section 5 presents the results of estimation using simulated data and flight data followed by a conclusion section.

Existing methods of aero database update

A brief description of a technique for determining the airplane model structure from flight data using splines and modified stepwise regression [5] is given below:

The general form of aerodynamic model equations can be written as:

$$Y(t) = \theta_0 + \theta_1 X_1 + \ldots\ldots + \theta_{q-1} X_{q-1} + \varepsilon$$

In the above equation $Y(t)$ represents the resultant coefficient of arrodynamic force or moment (the dependent variable), θ_0 to θ_{q-1} are the constants in spline representation of the aerodynamic functions, and X_1 to X_{q-1} are the airplane response and input variables and their combinations (the independent variables).

Assuming that a sequence of N observations of Y and of X has been made, then an adequate model for the aerodynamic coefficients can be determined by applying the stepwise regression. When formulating spline functions in expressions for an aerodynamic coefficient, the degree of spline and the number and location of knots must be specified.

Another method used at BCAG utilizes information measured on board the flight-test aircraft and compares the forces and moments required to satisfy the airplane equations of motion with those computed by the existing simulation [6]. The differences between the required and computed aerodynamic coefficients are used to update the simulator aerodynamic database. It is assumed that the required total aerodynamic coefficient for each degree of freedom can be represented as the value computed by the exsiting simulation for the given aircraft state plus a difference coefficient, i.e.,:

$$\begin{aligned}
C_{X_{aero}} &= C_{X_{sim}} + \Delta C_X \\
C_{Y_{aero}} &= C_{Y_{sim}} + \Delta C_Y \\
C_{Z_{aero}} &= C_{Z_{sim}} + \Delta C_Z \\
C_{l_{aero}} &= C_{l_{sim}} + \Delta C_l \\
C_{m_{aero}} &= C_{m_{sim}} + \Delta C_m \\
C_{n_{aero}} &= C_{n_{sim}} + \Delta C_n
\end{aligned}$$

A work on identification of a nonlinear aerodynamic model of the F-14 addresses subset regression approach for determining model structure and generalized filter error method for parameter estimation to prove the model validity [7].

The identification of DO-328 aerodynamic database for a level D flight simulator utilizes postulations of suitable derivative models in an analytical form [8].

In the present work. a technique that directly updates the aerodynantic coefficient dependency on one or more independent variables represented in table lookup form is proposed. This technique can encompass model structure determination using splines and modified regression techniques. For the parameter estimation RLS is used in this technique. The motivation is to utilize this technique in real time. The novelty in this technique is that of obtaining non-linear aerodynamic functions in the table look-up form

2. Aircraft Model

To demonstrate the application of the proposed technique, the short period model of the aircrafi is considered. The approximate linear short period model of aircrafi assumes the following form:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & Z_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta e} \\ M_{\delta e} \end{bmatrix} \delta e \qquad (1)$$

During large amplitude maneuvers, the aircraft reponses like angle of attack and pitch rate are affected by nonlinearities. For the purpose of demonstrating the novel estimation technique, we postulated a nonlinear short period model as follows:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z(\alpha) & Z_q \\ M(\alpha) & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta e} \\ M_{\delta e} \end{bmatrix} \delta e \qquad (2)$$

where $Z(\alpha)$ and $M(\alpha)$ are vertical force and pitching moment as nonlinear functions of angle of attack. The function dependency on two or more variables can also be accommodated. For e.g.,

$$\begin{aligned}
\dot{\alpha} &= Z(\alpha, \delta e) + Z_q q \\
q &= M(\alpha, \delta e) + M_q q
\end{aligned} \qquad (3)$$

and

$$\begin{aligned}
\dot{\alpha} &= Z(\alpha, q, \delta e) \\
\dot{q} &= M(\alpha, q, \delta e)
\end{aligned} \qquad (4)$$

In the present work, the model given in eq.(2) has been used, although it is possible to apply the proposed technique to models represented by eqs.(3) and (4). The later models require larger data set to properly capture the joint dependence on more than one variable.

3. Linear Interpolation

The linear table lookup method is a common method of representing the aerodynamic nonlinearities of an aircraft. The choice of breakpoints is important in capturing the variation in the coefficient. The usual independent variables are angle of attack, sideslip and control surface deflection. The rate dependence becomes important for rotary balance data at high rotation rates. For a typical aircraft it is common to have joint dependence on more than one variable leading to a multi-dimensional table lookup. We motivate the estimation procedure by considering linear interpolation.

Consider the one-dimensional interpolation formulae:

$$\begin{aligned} y(x) &= y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)} \cdot (x - x_1) \\ &= \frac{(x_2 - x)}{(x_2 - x_1)} \cdot y_1 + \frac{(x - x_1)}{(x_2 - x_1)} \cdot y_2 \\ &= W_l(x) \cdot y_1 + W_r(x) \cdot y_2 \\ x &\in [x_1, x_2] \end{aligned} \tag{5}$$

This equation gives the function value between two breakpoints (x_1, y_1) and (x_2, y_2). We have recast the formulae in the form of a left weight $W_l(x)$ multiplying the left hand value of the function y_1 and a right weight $W_r(x)$ multiplying the right hand value y_2. It is noted that the weights are themselves functions of the independent variable x. The value of the function $y(x)$ is a linear combination of the weights and the values at breakpoints. The advantage of expressing in this manner is that the linear interpolation can be generalized to multiple dimensions easily. Consider the two dimensional interpolation formulae (Fig. 1):

$$\begin{aligned} y(\alpha, \beta) &= W_{l1}(\alpha) \cdot W_{l2}(\beta) \cdot y_{i,j} \\ &+ W_{l1}(\alpha) \cdot W_{r2}(\beta) \cdot y_{i,j+1} \\ &+ W_{r1}(\alpha) \cdot W_{l2}(\beta) \cdot y_{i+1,j} \\ &+ W_{r1}(\alpha) \cdot W_{r2}(\beta) \cdot y_{i+1,j+1} \\ \alpha &\in [\alpha_i, \alpha_{i+1}], \beta \in [\beta_j, \beta_{j+1}] \end{aligned} \tag{6}$$

The weights in the above equations are computed separately for each dimension in an identical manner to the one-dimensional interpolation in equation (5). To compute the value of the function, the four possible combinations of the two one-dimensional weights are formed and multiplied with the corresponding breakpoint value of the two-dimensional function (equation (6)). This process can be followed for any higher dimensional table lookup with linear interpolation. In the context of the estimation problem posed in this paper, the values of dependent variable are the unknowns (y_{ij}). It is noted that the value of the function at an intermediate point $y(\alpha, \beta)$ is a linear function of the weights and the unknowns y_{ij}. This is true in general for the multi-dimensional linear interpolation scheme as well.

4. Estimation Procedure

The estimation problem for model structure given in eq.(2) consists of determining the unknown functions $Z(\alpha)$ and $M(\alpha)$ and the parameters $Z_q, M_q, Z_{\delta e}$ and $M_{\delta e}$. The estimation of unknown (possibly nonlinear) functions $Z(\alpha)$ and $M(\alpha)$ is simplified by treating the function values at selected breakpoints as the unknowns and assuming linear interpolation for values between the breakpoints. For example, if 'n' equidistant breakpoints $(\alpha_1, \alpha_2, \ldots, \alpha_n)$ in the independent variable α are chosen and the corresponding function values for $M(\alpha)$ are represented as $M_1, M_2, \ldots, M_n$, the vector of unknowns can be written as:

$$\beta_1 = \begin{vmatrix} M_1 \\ . \\ . \\ . \\ M_n \\ M_q \\ M_{\delta e} \end{vmatrix} \tag{7}$$

Consider the kth data point of the time response. Using the linear interpolation formulae, the pitching moment dynamics for the model in eq.(2) can be written as:

$$\dot{q}_k = \begin{bmatrix} 0 & . & . & W_{lk} & W_{rk} & . & . & 0 & q_k & \delta e_k \end{bmatrix} \beta_1 + e_k \tag{8}$$

where W_{lk} and W_{rk} are the weights obtained from the interpolation scheme for the ith and i+1th breakpoints (i.e., for α lying between ith and i+1th breakpoints). All other locations will be zeros excluding the q_k and δe_k. The model error is represented by e_k

The above equation can be extended in the vector form for all 'm' data points and can be written as:

$$Y_1 = X^T \beta_1 + e \tag{9}$$

Similar argument holds good for angle of attack dynamics represented by eq.(2) and it is written as:

$$Y_2 = X^T \beta_2 + e \tag{10}$$

where

$$\beta_2 = \begin{bmatrix} Z_1 \\ . \\ . \\ . \\ Z_n \\ Z_q \\ Z_{\delta e} \end{bmatrix} \qquad (11)$$

Note that if the aoa breakpoints are same for pitching moment dynamics and angle of attack dynamics, the regressor X is same as shown by eqs.(9) and (10). In general, the breakpoints need not be same for multiple unknown functions and they need not be equidistant. The technique allows the user to choose these depending on the problem at hand.

In the present study, the regressor is assumed to he same for both state equations represented in eq.(2). Hence. the problem can be further simplified by simultaneously estimating β_1 and β_2 as β

$$\text{i.e., } \beta = \begin{bmatrix} Z_1 & M_1 \\ . & . \\ . & . \\ Z_n & M_n \\ Z_q & M_q \\ Z_{\delta e} & M_{\delta e} \end{bmatrix} \qquad (12)$$

where n is the number of angle of attack break points assumed. The common regressor X takes the following form:

$$X = \begin{bmatrix} 0 & . & 0 & W_{l1} & W_{r1} & . & . & 0 & q_1 & \delta e_1 \\ W_{l2} & W_{r2} & 0 & . & . & 0 & 0 & 0 & q_2 & \delta e_2 \\ 0 & W_{l3} & W_{r3} & . & . & 0 & 0 & 0 & q_3 & \delta e_3 \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & W_{lm} & W_{rm} & . & . & q_m & \delta e_m \end{bmatrix} \qquad (13)$$

arid output Y is written as:

$$Y = \begin{bmatrix} \dot{\alpha}_1 & \dot{q}_1 \\ \dot{\alpha}_2 & \dot{q}_1 \\ . & . \\ . & . \\ \dot{\alpha}_m & \dot{q}_m \end{bmatrix} \qquad (14)$$

The unknown β is estimated using

$$\beta = (X^T X)^{-1}(X^T Y) \qquad (15)$$

Using the matrix inversion lemma, the recusive form of eq.(3) can he obtained which results in recursive least squares (RLS) estimation. The unknown parameter vector β can be estimated recursively using the following steps:

Initialize the algorithm by setting $P_0 = \delta^{-1} I$, where δ is a small positive constant and

$$\hat{\beta}_0 = 0 \qquad (16)$$

For each instant of time, $n = 1, 2, \ldots, N$ compute

$$\begin{aligned} \pi_n &= X_n P_{n-1} \\ \kappa_n &= 1 + \pi_n X_n^T \\ k_n &= P_{n-1} X_n^T / \kappa_n \\ \alpha_n &= Y_n - (\hat{\beta}_{n-1} X_n^T)^T \\ \hat{\beta}_n &= \hat{\beta}_{n-1} + (k_n \alpha_n)^T \\ P'_{n-1} &= k_n \pi_n \\ P_n &= (P_{n-1} - P'_{n-1}) \end{aligned} \qquad (17)$$

Where P is the correlation matrix. The advantage of recursive estimation lies in avoiding the matrix inversion and achieving computational simplicity. At every 'n' the regressor (X) will have only four non zero elements (for estimating the model described by eq.(2)) and hence other zero elements can be dropped from X. The covariance matrix can also be made to have reduced dimension. This achieves reduced number of floating-point operations and the running time of the algorithm is made faster. Also it may be used to perform real time estimation.

5. Results

Initially lo show the validity of the technique, simulated data is used. While simulating the aircraft responses, non-linearity is introduced by means of incorporating $Z(\alpha)$ and $M(\alpha)$ in the model. It is iniplemented in the form of table look-up using realistic values obtained from flight estimates. The idea is to bring out the validity of the technique. Also the input used in the simulation to excite the plant is that of a real flight's large amplitude maneuver control surface input. The angle of attack break points are assumed at 1 deg interval covering the minimum and maximum values for that maneuver. At each breakpoint, the nonlinear function values $Z(\alpha)$ and $M(\alpha)$ are estimated. In order to test the robustness of the technique for measurement noise, gaussian noise of SNR = 30 is added with the simulated time histories. Forward and reverse digital filtering is used to remove the noise. After the noise removal, the data is used in the estimation. The estimated $Z(\alpha)$ and $M(\alpha)$ comparison with their true values can be seen from figures (2) and (3). The comparison of estimated time histories with simulated time histories is shown in fig.(4). There is a slight mismatch seen in the comparison plots. This is because of the noisy data, but still it is acceptable. In a closed loop aircraft, data correlations can hamper the identification. Hence this technique is tested for a longitudinally unstable closed loop aircraft. The mathematical model of the unstable aircraft is given below:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{u}/u_0 \end{bmatrix} = \begin{bmatrix} Z_\alpha & 1 & 0 & Z_{u/u_0} \\ M(\alpha) & M_q & 0 & M_{u/u_0} \\ 0 & 1 & 0 & 0 \\ X_\alpha & 0 & X_\theta & X_{u/u_0} \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \\ u/u_0 \end{bmatrix} + \begin{bmatrix} Z_{\delta e} \\ M_{\delta e} \\ 0 \\ X_{\delta e} \end{bmatrix} \delta_e$$

The simulink block diagram of the aircraft along with the controller is shown in fig. (5). The nonlinearity was introduced in $M(\alpha)$. The estimation was performed to estimate the short period characteristics of aircraft. The data was corrupted with measurement gaussian noise of SNR = 30. The comparison of estimated $M(\alpha)$ is shown against its true values in fig. (6). The comparison looks satisfactory. The comparison of estimated time histories is shown against the simulated time histories in fig. (7). The fit is satisfactory.

Subsequent to the validation of the technique using simulated data, it is tested with flight data. The aoa and pitch rate time histories are not affected by measurement noise significantly. Hence no filtering is used. The estimated values of $Z(\alpha)$ and $M(\alpha)$ are given in figures (8) and (9). The comparison of estimated time histories with flight responses is shown in fig (10). The RLS was also found to yield the same estimates as least squares for flight data but with lower computations as matrix inversion is avoided and also it may be implemented in real time.

6. Conclusions

A novel technique: Direct Update of Nonlinear Aerodynamic Coefficients (DUNAC) is proposed for updating aerodynamic database for coefficients in the form of table-look-up. The technique is validated against simulated data for its functionality and also tested for flight data. The technique can be extended for estimation of nonlinear functions of two or more variables.

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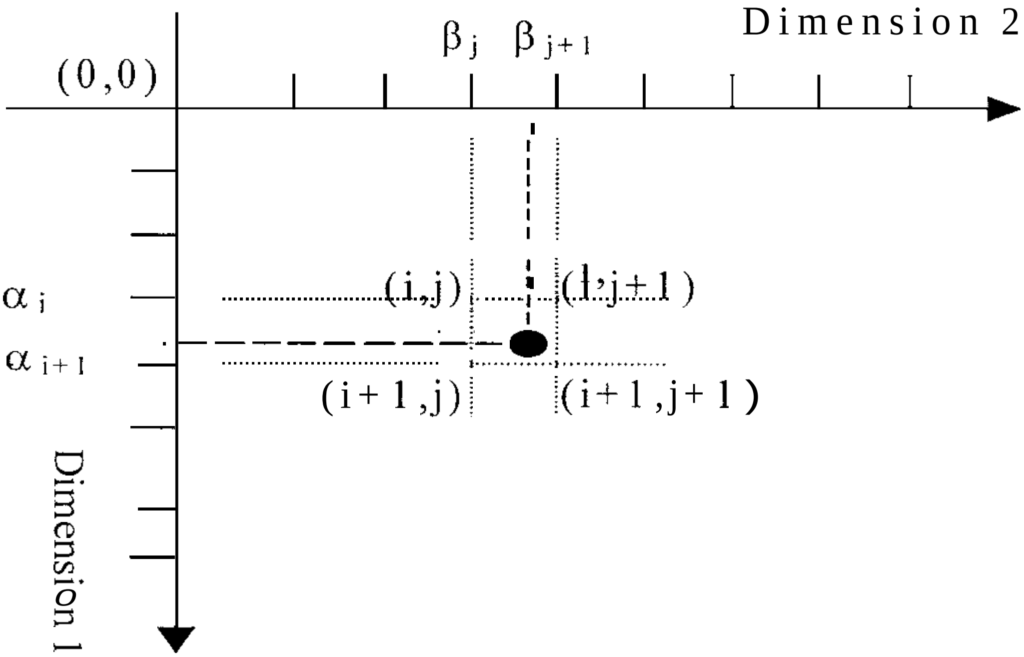


Figure 1 Two-dimensional interpolation grid

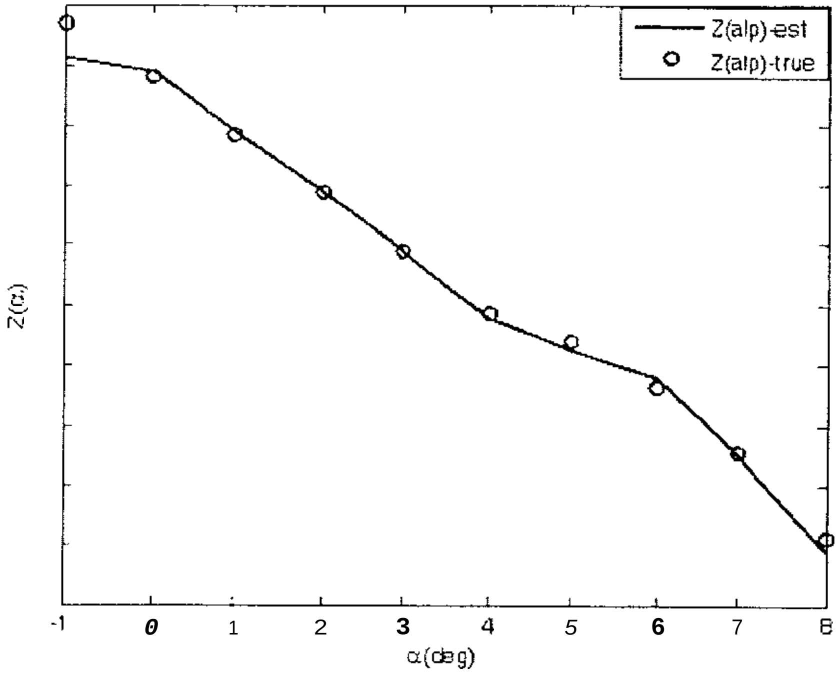


Figure 2 Match of estimated $Z(\alpha)$ with its true values

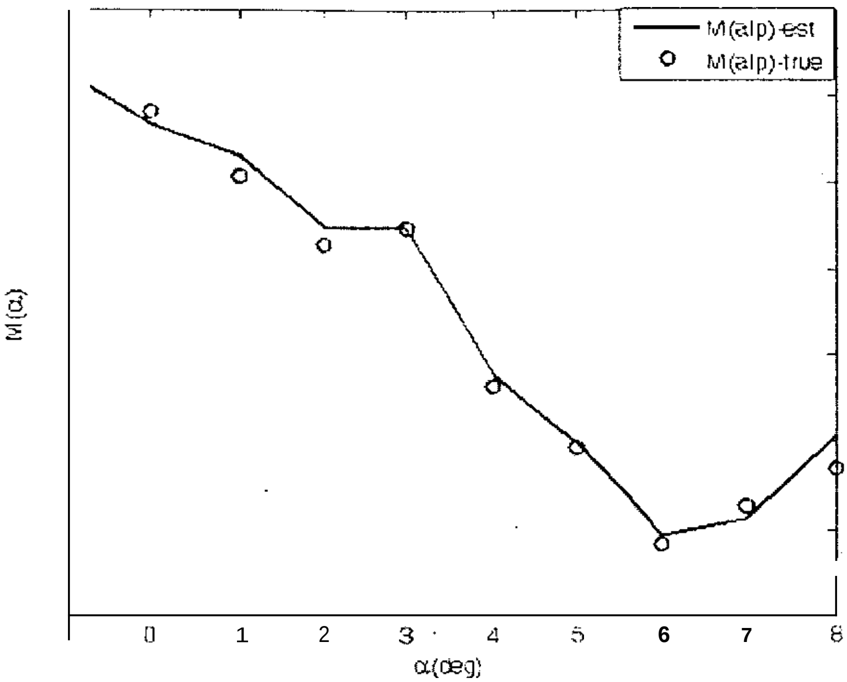


Figure 3 Match of estimated M(α) with its true values

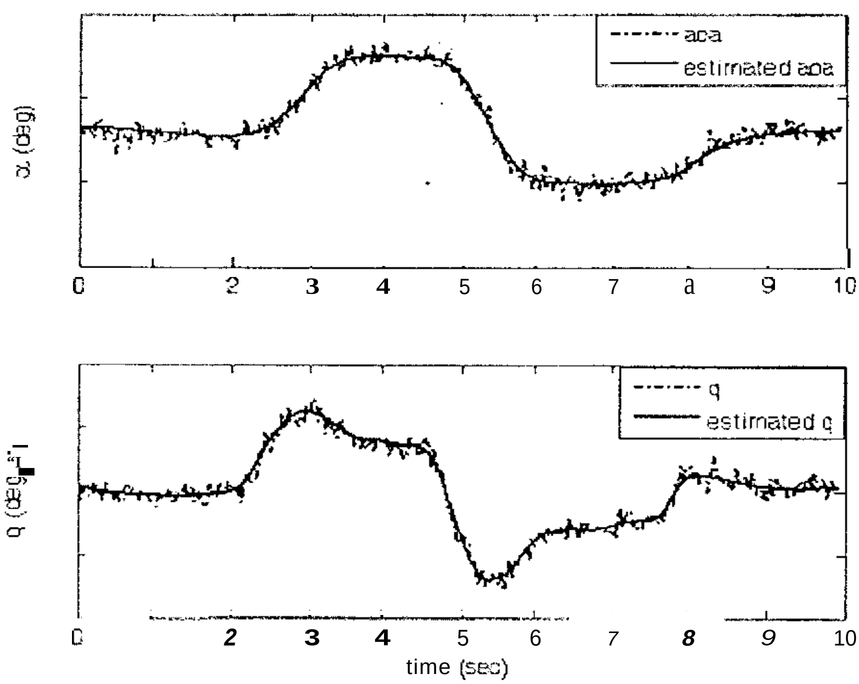
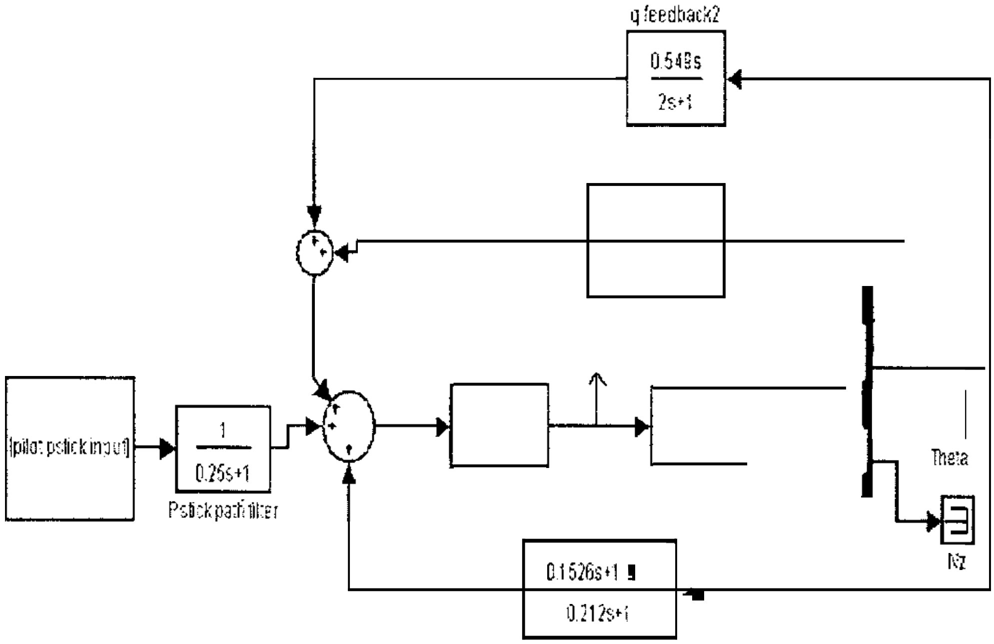


Figure 4 Match of estimated time histories with its true values-Noisy data



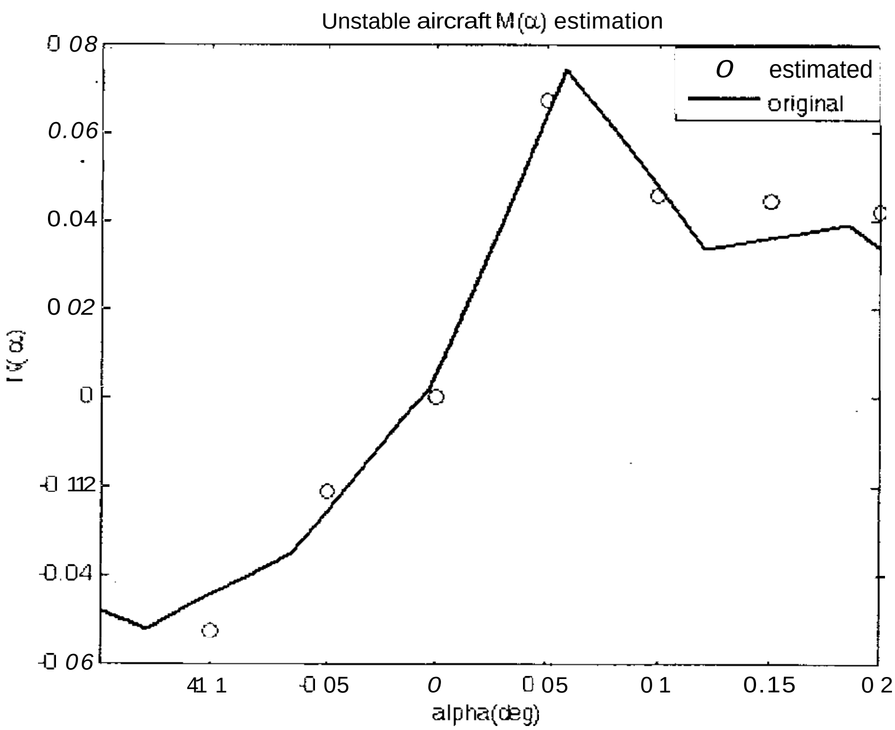


Figure **6** Match of estimated M(α) with its true values-unstable aircraft

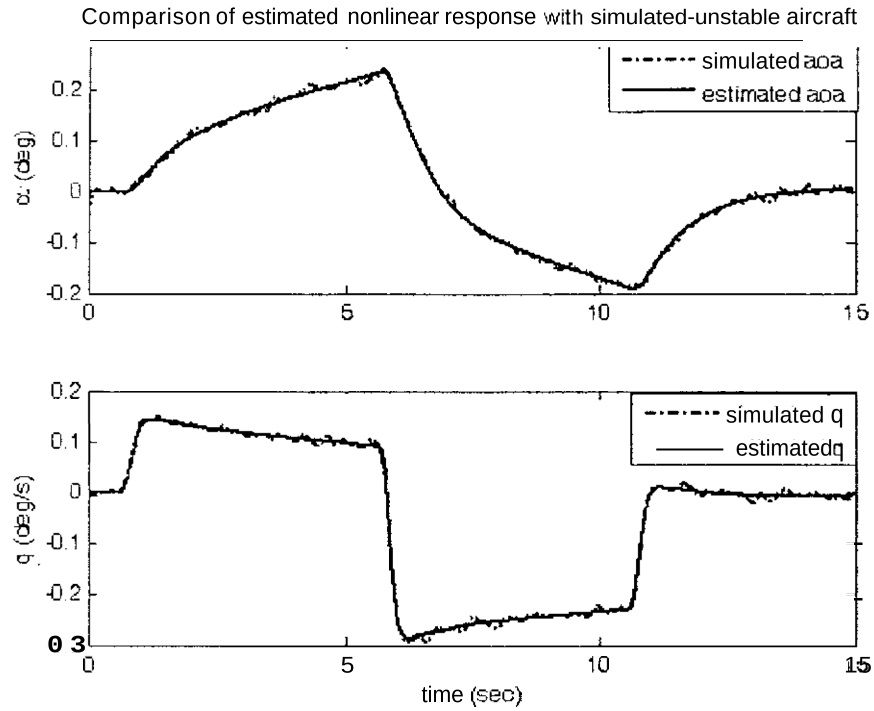


Figure 7 Match of estimated time histories with its true values-Noisy data-unstable aircraft

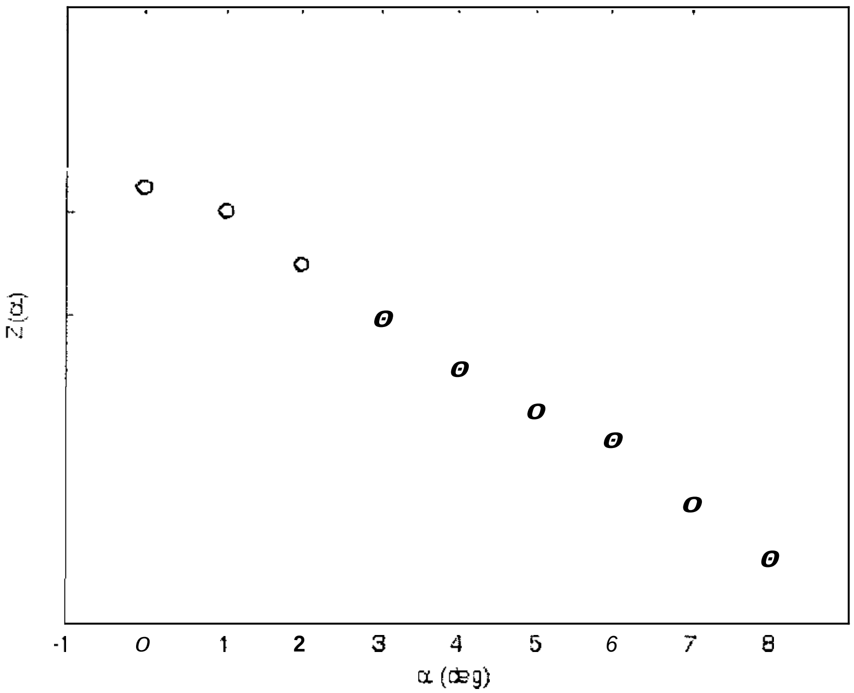


Figure 8 $Z(\alpha)$ estimates from flight data

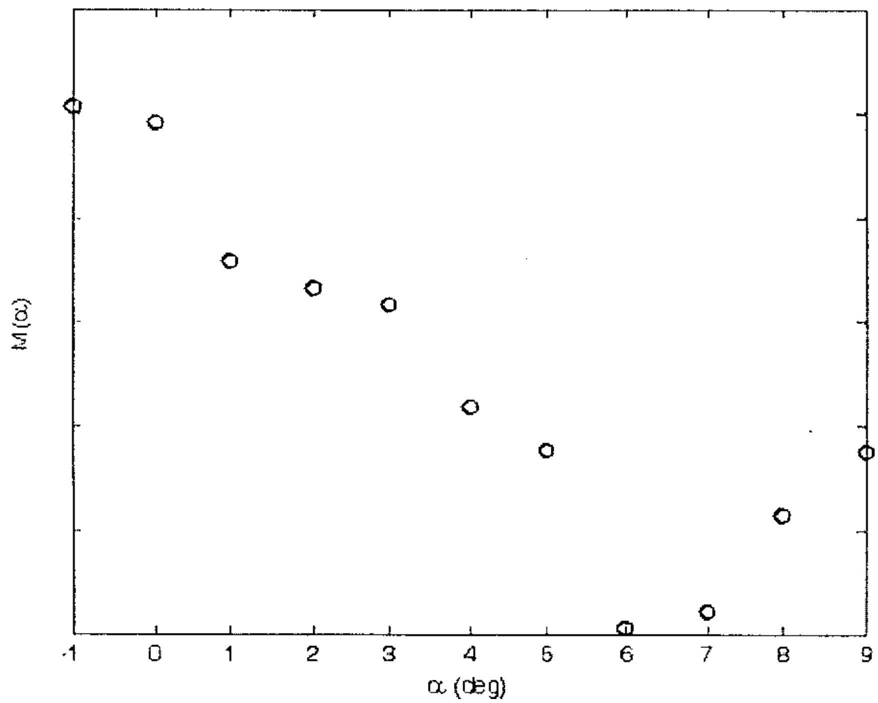


Figure 9 M(α) estimates from flight data

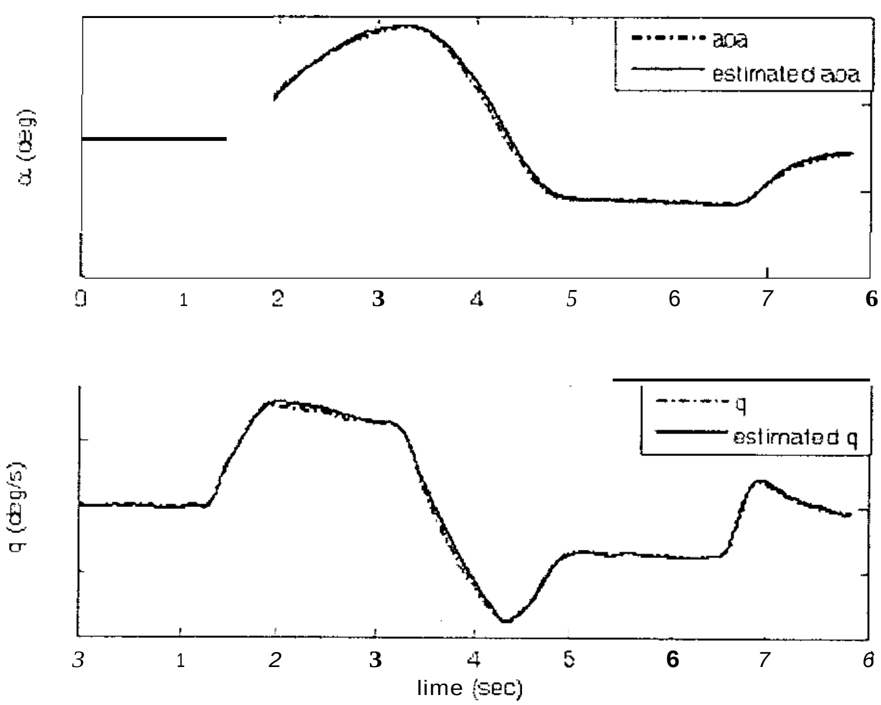


Figure 10 Match of estimated time histories with flight data